Math 53: Multivariable Calculus **Sections 212, 213**

Worksheet for 2020-09-14

Problem 1. Letting a be a fixed constant, consider the function

$$f(x,y) = \begin{cases} \dfrac{(x+y)^a}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Is f continuous if $a = 0$? If $a = 1$? If $a = 2$? If $a = 3$?

f is continuous away from $(0,0)$, so the question is just whether f is cont. @ $(0,0)$, i.e. if $\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0) = 0$.

If $a=0$: $f(x,y) = \dfrac{1}{x^2+y^2}$. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1}{x^2+y^2} = +\infty$ (i.e. DNE)

so f is not continuous.

$a=1$: try along any line and again find DNE.

$a=2$: $\displaystyle\lim_{x\to 0} \frac{(x+mx)^2}{x^2+m^2x^2} = \frac{(1+m)^2}{1+m^2}$. If $m=0$ this is 1, if $m=1$ this is 2,

so limit DNE.

$a=3$: The limit exists and is zero (switch to polar and squeeze for instance, as done in discussion example) so f is continuous.

Problem 2. Consider $x^2 + y^2 + z^2 = 1$. Note that $\frac{\partial x}{\partial y}$ for example means to view the equation as implicitly defining x as a function of y and z, and then to take the partial derivative of that function with respect to y (i.e. treating z as constant).

Compute the quantities $\frac{\partial x}{\partial y}$, $\frac{\partial y}{\partial z}$, $\frac{\partial z}{\partial x}$, and then compute their product (notice that the answer is *not* 1, giving you an example for why you should not think of these expressions as "fractions").

$2x\frac{\partial x}{\partial y} + 2y = 0$ so $\frac{\partial x}{\partial y} = -\frac{y}{x}$.

$2y\frac{\partial y}{\partial z} + 2z = 0$ so $\frac{\partial y}{\partial z} = -\frac{z}{y}$

$2x + 2z\frac{\partial z}{\partial x} = 0$ so $\frac{\partial z}{\partial x} = -\frac{x}{z}$.

The product is -1.